

Analysis of Discontinuities of Microstrip and Suspended Substrate Lines

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Based on an improved formulation of the singular integral equation method for microstrip and suspended substrate lines, their discontinuities including arbitrary composition of steps are analyzed using the mode matching method. Results show that previous analyses using the mode matching were not sufficiently accurate. The influence of complex modes is discussed.

**Introduction**

The rigorous analysis of discontinuities for microstrip and suspended substrate lines is receiving increasing interests in recent years. Although this effort has been made by many authors using various methods [1]-[9], the problem seems to be still not completely solved. First of all, their results showed a lack of good agreement with each other, which should not have happened if each had solved the problem rigorously. Even for a single step discontinuity the difference is unfortunately not small, especially if phase angles of S-parameters were concerned. Thus the true results for discontinuities of microstrip-like lines still need to be identified. Furthermore, very little information is available on typical passive elements used in practice.

The present analysis is based on an improved formulation of the singular integral equation (SIE) method. It permits rigorous and efficient solution of a large number of modes, say 100. The mode matching (MM) method is then applied to analyse the discontinuities. A corresponding computer programme has been developed to deal with cascaded discontinuities. At present it includes arbitrary composition of step discontinuities.

In previous publications using the MM method the number of modes that can be accurately determined is usually quite limited. For example, no more than 10 were computed in [5]-[7]. Besides most of them except [7] have not considered complex modes, which may cause serious errors as is shown later. As a consequence numerical results presented were generally not satisfactory and their convergence was not guaranteed or proved.

**Theory**

The accuracy of the MM method depends basically on the accuracy of the solution of eigen modes. This can be achieved with the improved formulation of the SIE method for microstrip and suspended substrate lines. Not only the convergence of the main characteristic equations but also that of the vanishing conditions that must be imposed on the tangential field and current components  $E_z$  and  $J_x$  in the strip plane are accelerated by making use of their asymptotic expressions. Thus each of the equations comprising the final characteristic matrix possesses a higher convergence rate in terms of the series truncation order. Due to the complicated mathematical derivation, the characteristic matrix in [7], [11] and [12] was only available in a small size, which is not enough if many modes need to be calculated. We have succeeded in finding out some recursion relations, so that it is possible to calculate the matrix elements to any given matrix size. Furthermore no numerical integration as in [7], [11] and [12] is necessary here, all expressions are given in analytic forms. The characteristic matrix is reconstructed so that its determinant is an analytic function in the whole complex plane. By making use of fine properties of complex analytic functions the search for complex roots becomes much easier. Complex modes can be proved to exist only in the region  $\text{real}(\beta^2) < 0$ .

Based on the above efficient determination of the guided modes at both sides of the discontinuity the MM method has been applied. The relative convergence problem is not critical in the present case. Since the strip thickness is assumed to be zero, the two lines that form a discontinuity have the same cross sectional area. It can be proved theoretically that only an equal number of modes at both sides gives generally reasonable results. One should also notice that both modes of a complex pair must be taken into account. If only one is included in the analysis the energy conservation will be destroyed, which may cause, for example, magnitudes of S-parameters to be greater than one.

**Results and Discussions**

Table 1 shows the convergence behaviour of a single step. Results are given at a low frequency for a weak discontinuity, so that only several modes are enough to get accurate results. The results pre-

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sented in [5] are also given. It is easily seen that our results show a better convergence.

Comparison is also made with other authors. For the step shown in Fig.3.2.1.2 of [6] 10,20,30,40,50 and 60 modes have been used here to study the accuracy of the computation there. At least 30 modes are necessary for the whole frequency range concerned. [6] used only about 10 modes. Although his results provide with some accuracy information about the magnitudes at lower frequencies, the error for phase angles is obvious and  $\Phi_{21}$  has even a wrong sign. [7] used also the SIE method for the modes determination. However, only a  $2 \times 2$  characteristic matrix was applied, which is, according to our analysis, not enough. Furthermore the power orthogonality relation that is no longer valid for complex modes has been used there.

Fig.2 shows that ignoring the existence of complex modes may cause serious errors.

Fig.3 shows the insertion loss of a four-section quarter-wave impedance transformer ( $10\Omega$ - $50\Omega$ ) with a designed bandwidth 80% centring at 10GHz.

Several seven-section lowpass filters on a substrate with  $\epsilon_r=6.15$  and  $d=0.635\text{mm}$  (RT/duroid 6006) have been analysed using 80 modes. The designed maximum attenuation in the pass band ( $L_{\text{ar}}$ ) is 0.1db, 0.2db, 0.5db, 1.0db, 2.0db and 3.0db respectively. The designed band edge frequency  $f_1$  is 12GHz. Both forms, namely the form beginning with an inductive section (form 1) and its dual form beginning with a capacitive section (form 2) have been taken into account (see Fig.4). Following conclusions can be obtained: (i) consideration of higher order modes makes the calculated  $f_1$  much smaller than designed. Their difference can be as large as about 1.5GHz. So the design method based on quasi TEM analysis needs to be improved. Complex modes have a stronger influence on  $f_1$  than other non-complex higher order modes. Without considering complex modes the error would be more than 10db in the stop band. (ii) The lower design values ( $L_{\text{ar}} \leq 1\text{db}$ ) can not be realized even in the reduced pass band. However the larger values ( $L_{\text{ar}}=2\text{db}$  and 3db) can be well achieved in the reduced pass band. (iii) Filters with form 2 are better approximations to the designed equal-ripple behavior than their dual ones. The difference of unequal ripples is smaller with form 2 and the calculated  $f_1$  is al-

ways larger than that of their dual form. So form 2 is more recommended. Results for two cases are given in Fig.5 and Fig.6.

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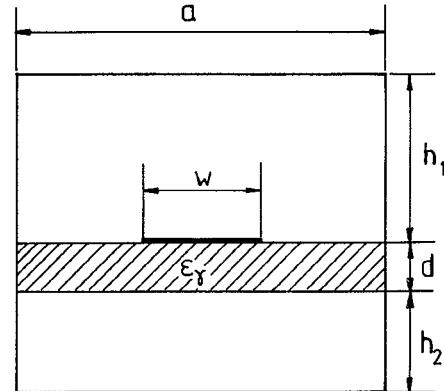
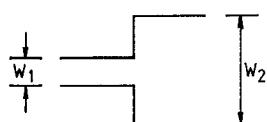


Fig.1 Cross section view of the transmission line

Table 1 S-parameters of a step discontinuity versus number of expansion modes.  $\epsilon_r=2.2$ ,  $f=4\text{GHz}$ ,  $a=3.2$ ,  $h_1=1.27$ ,  $d=0.127$ ,  $h_2=0$ ,  $w_1=0.381$ ,  $w_2=2w_1(\text{mm})$

N	$ S_{11} $	$\Phi_{11}$	$ S_{21} $	$\Phi_{21}$	$S_{11}$ [5]	$S_{21}$ [5]
6	0.2470453	180.0131	0.9690039	179.9939	(-0.1729, $1.346 \times 10^{-4}$ )	(0.9849, $-8.359 \times 10^{-5}$ )
10	0.2470446	180.0215	0.9690041	179.9891	(-0.2706, $-1.73 \times 10^{-4}$ )	(0.9627, $-2.08 \times 10^{-5}$ )
20	0.2470462	179.6721	0.9690037	179.9085		
30	0.2470490	179.5222	0.9690030	179.8802		
40	0.2470458	179.5468	0.9690038	179.8745		
50	0.2470382	179.6570	0.9690057	179.8864		
60	0.2470354	179.6537	0.9690065	179.8726		



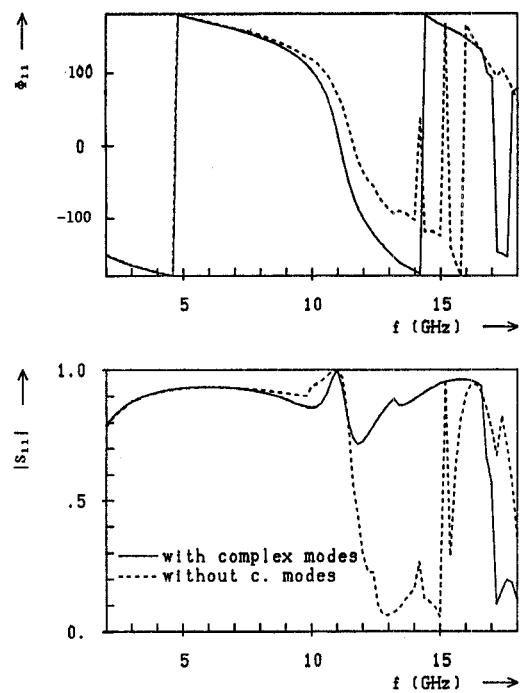


Fig.2 Influence of complex modes on a double step discontinuity.  $\epsilon_r=9.0$ ,  $h_1=5.08$ ,  $d=1.27$ ,  $h_2=0.$ ,  $a=9.52$ ,  $w_1=0.5$ ,  $w_2=8.0$ ,  $l=4.0$ (mm).

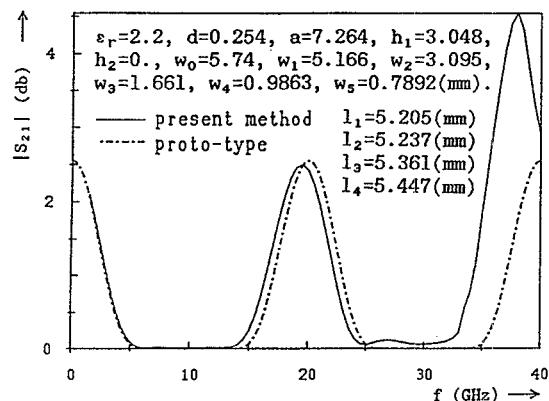
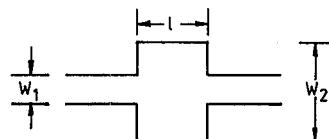
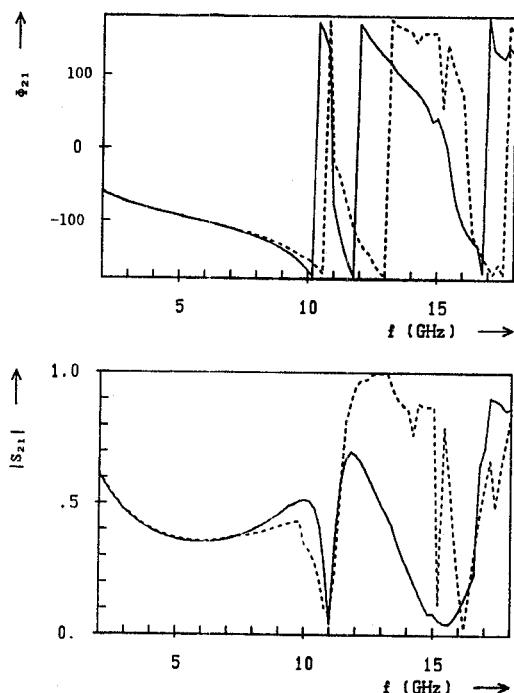
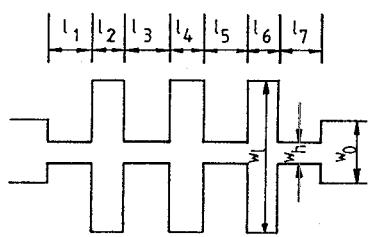
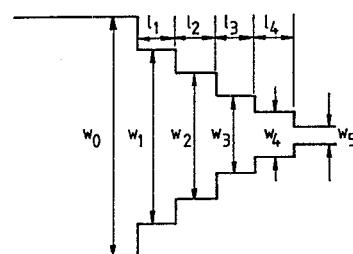
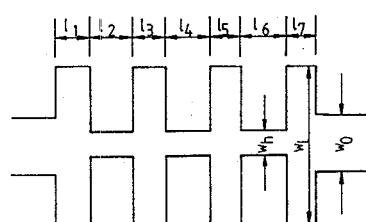


Fig.3 Attenuation of a four-section quarter-wave transformer.



(a) form 1



(b) form 2

Fig.4 Two forms of (seven-section) lowpass filters: (a) with first section being inductive; (b) with first section being capacitive

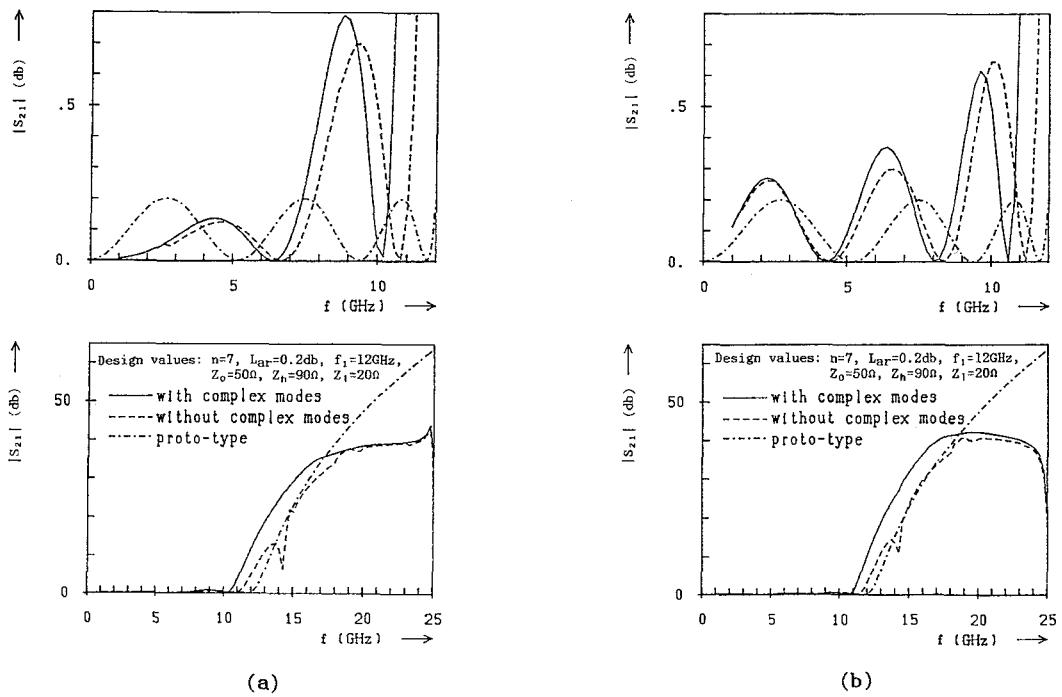


Fig.5 Attenuation of seven-section lowpass filters with designed  $L_{ar}=0.2\text{db}$ .  $w_0=0.9373$ ,  $w_h=0.2476$ ,  $w_1=3.617(\text{mm})$ ,  $h_1=10d$ ,  $h_2=0$ .  $a=w_1+10d$ . (a) form 1 :  $l_1=l_7=1.285$ ,  $l_2=l_6=0.8933$ ,  $l_3=l_5=1.779$ ,  $l_4=0.9583(\text{mm})$ . (b) form 2 :  $l_1=l_7=0.8901$ ,  $l_2=l_6=1.289$ ,  $l_3=l_5=1.309$ ,  $l_4=1.371$

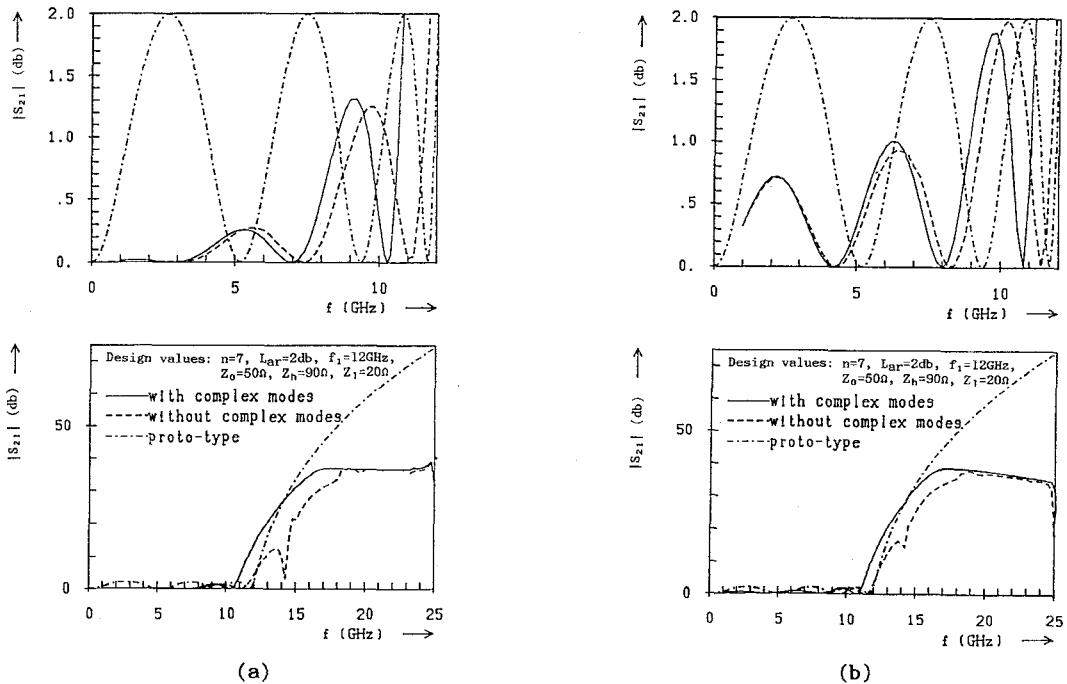


Fig.6 Attenuation of seven-section lowpass filters with designed  $L_{ar}=2\text{db}$ .  $w_0=0.9373$ ,  $w_h=0.2476$ ,  $w_1=3.617(\text{mm})$ ,  $h_1=10d$ ,  $h_2=0$ .  $a=w_1+10d$ . (a) form 1 :  $l_1=l_7=1.992$ ,  $l_2=l_6=0.6202$ ,  $l_3=l_5=2.241$ ,  $l_4=0.6461(\text{mm})$ . (b) form 2 :  $l_1=l_7=1.513$ ,  $l_2=l_6=0.9250$ ,  $l_3=l_5=1.770$ ,  $l_4=0.9610$